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Ion-Acoustic Wave in Relativistic Nonisothermal Plasma with Negative Ions

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We study solitary wave formation in a nonisothermal plasma with negative ions. When the ions are considered to be relativistic. The variations of the amplitude, width, and the phase velocity are obtained explicitly with respect to the ratio of the ion densities and the streaming velocity.

1. INTRODUCTION

The study of solitary waves occupies a central position in present-day plasma research. During the last two or three decades various papers have appeared dealing with different theoretical aspects of nonlinear plasma theory. But some physical phenomena, such as relativistic effects and the effect of a finite boundary, have gained importance only very recently. An initial attempt to study relativistic effects in solitons was that of Das and Paul (1985). Afterward other effects such as Landau damping (Roy Chowdhury *et al.*, 1988) and two-temperature effects (Roy Chowdhury *et al.*, 1990) were incorporated. The importance of relativistic effects were seen in studies of laser plasma interactions and large-amplitude waves in plasmas (Tsytovich, 1974; Bingham *et al.*, 1990). On the other hand the study of plasmas with negative ions is very important for the explanation of many astrophysical and laboratory events (Das and Karmakar, 1990; Tagare, 1986).

In this study we analyze the important situation of a relativistic nonisothermal plasma in the presence of negative ions. Here both types of ions are considered to be relativistic, one of which is negative, the mass ratio being Q. After obtaining the modified KdV equation, we analyze the variation of the amplitude and width of the solitary wave as a function of the streaming velocities and the ratio of the ion densities.

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2. FORMULATION

As usual we assume that a hydrodynamic description is possible so that the equations governing the plasma are

$$\left.\begin{aligned}\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial x}(n_\alpha u_\alpha)=0 \qquad (1)\\ \frac{\partial \bar{u}_\alpha}{\partial t}+u_\alpha\frac{\partial \bar{u}_\alpha}{\partial x}=-\frac{\partial \phi}{\partial x} \qquad (2)\end{aligned}\right\}\quad \text{positive ion}$$

$$\left.\begin{aligned}\frac{\partial n_\beta}{\partial t}+\frac{\partial}{\partial x}(n_\beta u_\beta)=0 \qquad (3)\\ \frac{\partial \bar{u}_\beta}{\partial t}+u_\beta\frac{\partial \bar{u}_\beta}{\partial x}=\frac{Z}{Q}\frac{\partial \phi}{\partial x} \qquad (4)\end{aligned}\right\}\quad \text{negative ion}$$

Z is the charge of the ion, and

$$\frac{\partial^2\phi}{\partial x^2}=n_e+Zn_\beta-n_\alpha \qquad (5)$$

where n_α, n_β, U_α, U_β are the densities and velocities for the two types of ions. n_e is the density of electrons,

$$n_e=1+\phi-\tfrac{4}{3}b\phi^{3/2}+\tfrac{1}{2}\phi^2+\cdots \qquad (6)$$

in the nonisothermal situation, and

$$\begin{aligned}\bar{U}_\alpha &\cong U_\alpha\left(1+\frac{u_\alpha^2}{2C^2}\right)\\ \bar{U}_\beta &\cong U_\beta\left(1+\frac{{U_\beta}^2}{2C^2}\right)\end{aligned} \qquad (7)$$

We start by stretching the coordinates as

$$\begin{aligned}\xi &=\varepsilon^{1/4}(x-\lambda t)\\ \tau &=\varepsilon^{3/4}t\end{aligned} \qquad (8)$$

and set

$$\begin{aligned}n_\alpha &=n_{\alpha 0}+\varepsilon n_{\alpha 1}+\varepsilon^{3/2}n\alpha_2+\cdots\\ n_\beta &=n_{\beta 0}+\varepsilon n_{\beta 1}+\varepsilon^{3/2}n_{\beta 2}+\cdots\\ u_\alpha &=u_{\alpha_0}+\varepsilon u_{\alpha_1}+\varepsilon^{3/2}u_{\alpha_2}+\cdots\\ u_\beta &=u_{\beta_0}+\varepsilon u_{\beta_1}+\varepsilon^{3/2}u_{\beta_2}+\cdots\\ \phi &=\varepsilon\phi_1+\varepsilon^{3/2}\phi_2+\varepsilon^2\phi_3+\cdots\end{aligned} \qquad (9)$$

where ε is a small parameter and these expansions satisfy the normalization condition

$$\begin{pmatrix} n_\alpha \\ n_\beta \\ u_\alpha \\ u_\beta \\ \phi \end{pmatrix} \rightarrow \begin{pmatrix} n_{\alpha 0} \\ n_{\beta 0} \\ u_{\alpha 0} \\ u_{\beta 0} \\ 0 \end{pmatrix} \quad \text{as} \quad x \rightarrow \pm\infty$$

Now using equations (8) and (9) in (1)–(5), we obtain, by equating like powers of ε,

$$\begin{aligned} n_{\alpha 1} &= \frac{n_{\alpha 0} u_{\alpha 1}}{\lambda - u_{\alpha 0}}; \qquad n\beta_1 = \frac{n\beta_0 u\beta_1}{\lambda - u\beta_0} \\ u_{\alpha 1} &= \frac{\phi_1}{(\lambda - u_{\alpha 0})(1 + 3u_{\alpha 0}^2/2C^2)} \\ \phi_1 &= n_{\alpha_1} - Zn_{\beta_1} \\ u_{\beta_1} &= \frac{Z/a\phi_1}{(\lambda - u_{\beta_1})(1 + 3u_{\beta 0}^2/2C^2)} \end{aligned} \tag{10}$$

whence we obtain the phase velocity as

$$\lambda = u_{\alpha 0} \pm \left(\frac{n_{\alpha 0}}{\gamma\alpha} - \frac{z^2}{a} \frac{n_{\beta 0}}{\gamma\beta} \right)^{1/2} \tag{11}$$

where

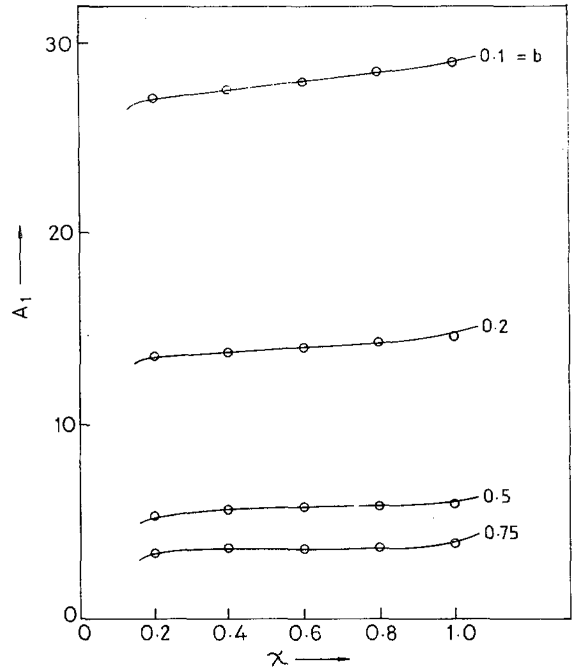
$$\gamma_\alpha = 1 + \frac{3u_{\alpha 0}^2}{2C^2}; \qquad \gamma_\beta = 1 + \frac{3u_{\beta 0}^2}{2C^2}$$

We now consider terms of higher order in ε, which leads to the following equations:

$$\frac{\partial^3 \phi_1}{\partial \xi^3} + 2b(\phi_1)^{1/2} \frac{\partial \phi}{\partial \xi} = \frac{\partial \phi_2}{\partial \xi} + z \frac{\partial n_{\beta 2}}{\partial \xi} - \frac{\partial n_{\alpha 2}}{\partial \xi} \tag{12}$$

$$\frac{\partial u_{\alpha 2}}{\partial \xi} = \frac{\lambda - u_{\alpha 0}}{n_{\alpha 0}} \frac{\partial n_{\alpha 2}}{\partial \xi} - \frac{1}{n_{\alpha 0}} \frac{\partial n_{\alpha 1}}{\partial \tau} \tag{13}$$

$$\frac{\partial u_{\beta 2}}{\partial \xi} = \frac{\lambda - u_{\beta 0}}{n_{\beta 0}} \frac{\partial n_{\beta 2}}{\partial \xi} - \frac{1}{n_{\beta 0}} \frac{\partial n_{\beta 1}}{\partial \tau} \tag{14}$$



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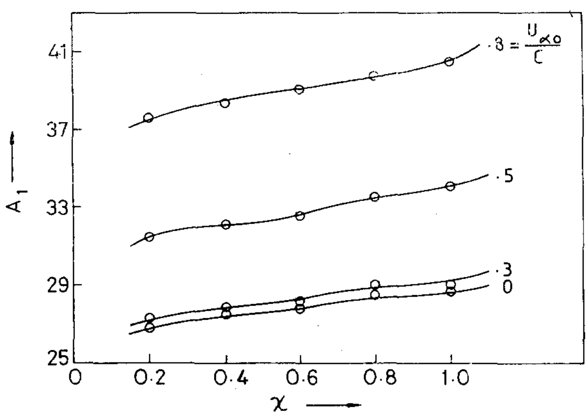
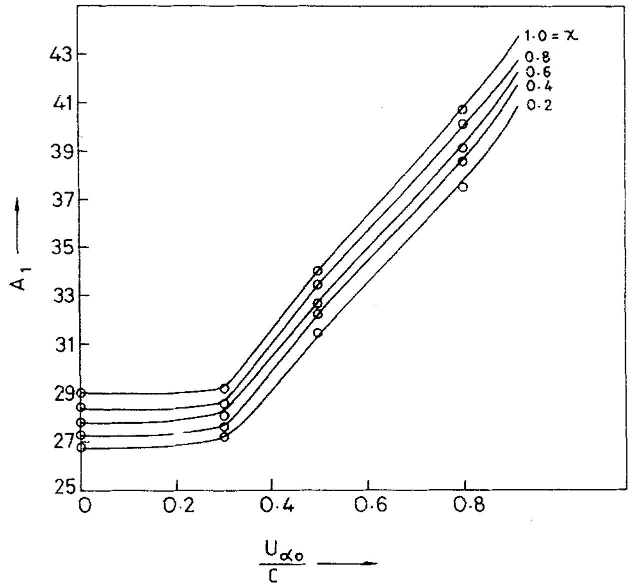


B

Fig. 1. (A) Plot of the amplitude of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$. Fixed $Q=6$, variable b, and fixed $u_{\alpha 0}=3\times 10^9$. (B) Plot of the amplitude of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$. Fixed $Q=6$, variable $U_{\alpha 0}/C$, and fixed $b=0.1$. (C) Plot of the amplitude of the soliton against the ratio $u_{\alpha 0}/C$. Fixed $Q=6$, fixed $b=0.1$, and variable $n_{\beta 0}/n_{\alpha 0}$. (D) Plot of the amplitude of the solution against the ratio $n_{\beta 0}/n_{\alpha 0}$. Fixed $u_{\alpha 0}=3\times 10^9$, fixed $b=0.1$, and variable Q.



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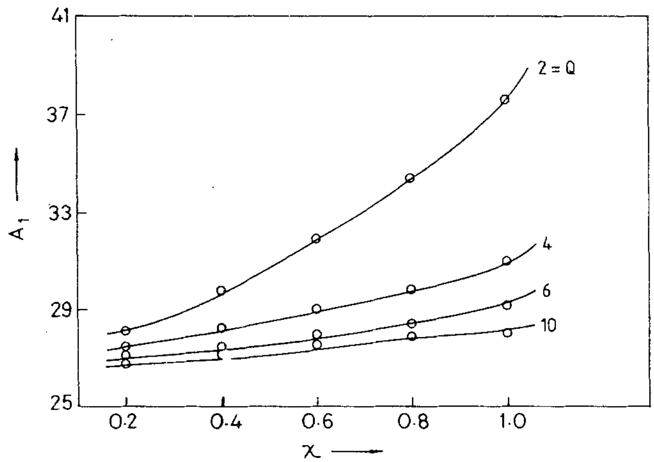

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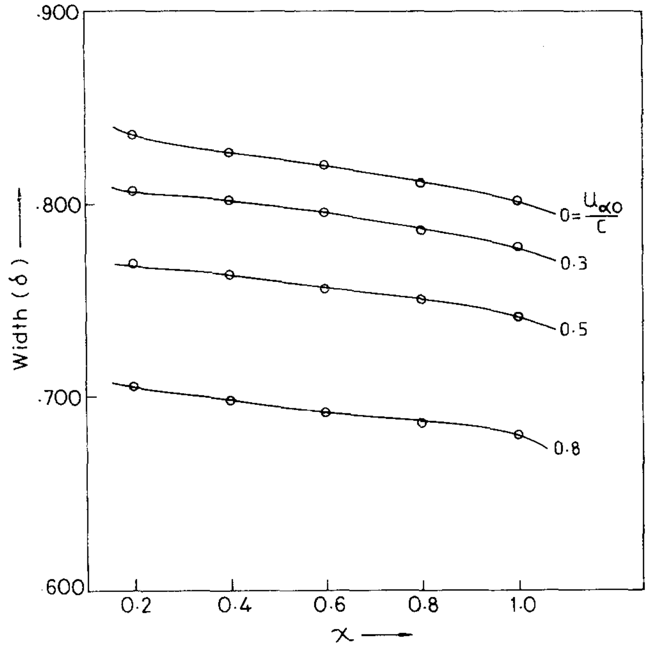

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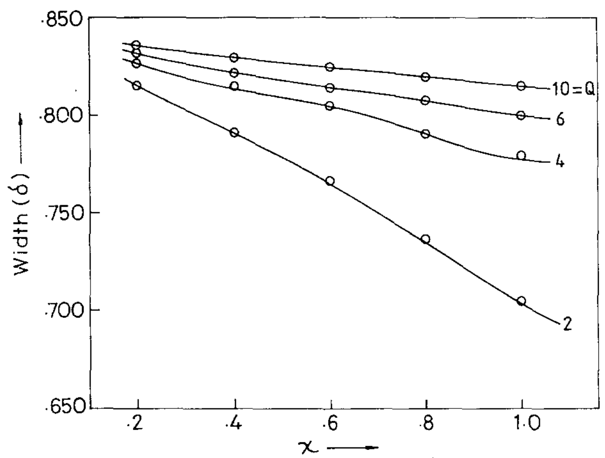


B

Fig. 2. (A) Plot of the width of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$. Fixed $Q=6$, variable $U_{\alpha 0}/C$. (B) Plot of the width of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$. Fixed $u_{\alpha 0}=3\times 10^9$, variable Q. (C) Plot of the width of the soliton against the ratio $U_{\alpha 0}/C$. Fixed $Q=6$, variable $n_{\beta 0}/n_{\alpha 0}$.

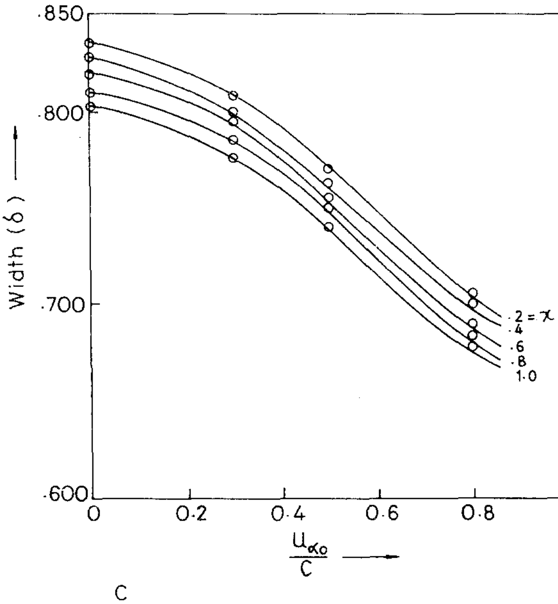


Fig. 2. Continued.

So eliminating the second-order quantities in favor of first-order ones, we get the modified KdV equation:

$$\frac{\partial \phi_1}{\partial \tau} + A(\phi_1)^{1/2} \frac{\partial \phi_1}{\partial \xi} + \beta \frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{15}$$

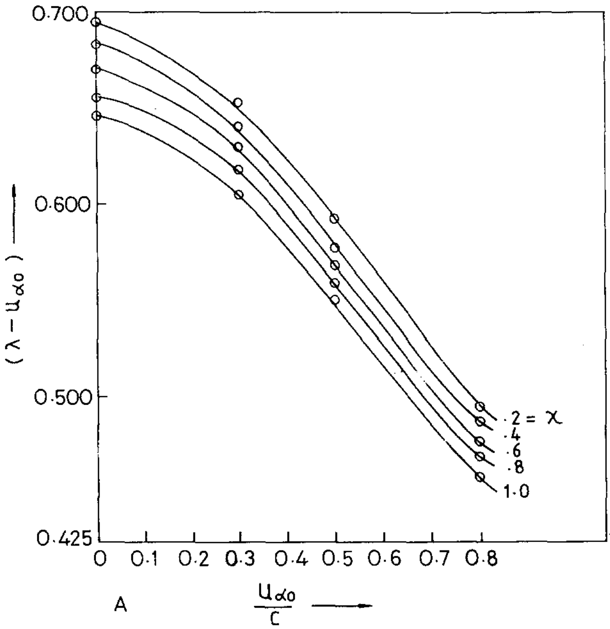
where

$$\begin{aligned} A &= \frac{2b}{R}, \qquad R = \frac{2}{(\lambda - u_{\alpha 0})^3} \left(\frac{n_{\alpha 0}}{\gamma_\alpha} - \frac{Z^2}{Q} \frac{n_{\beta 0}}{\gamma_\beta} \right) \\ B &= \frac{1}{S}, \qquad S = \frac{1}{(\lambda - u_{\alpha 0})^3} \left(\frac{2n_{\alpha 0}}{\gamma_\alpha} - \frac{2Z^2}{Q} \frac{n_{\beta 0}}{\gamma_\beta} \right) \end{aligned} \tag{16}$$

Solitary Wave Solution

Equation (15) is known to possess a solitary wave solution written as

$$\phi_1 = A_1^2 \operatorname{sech}^4(K\xi - \omega\tau) \tag{17}$$



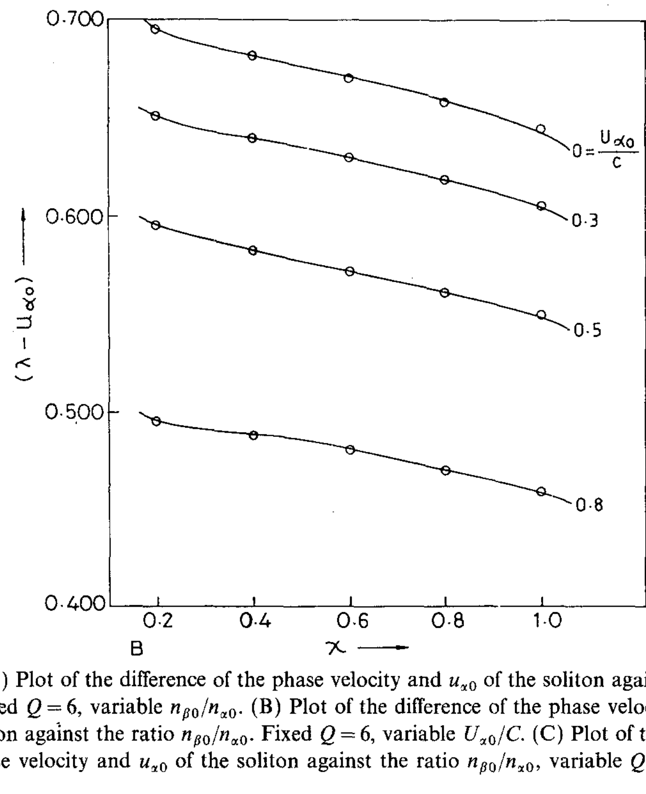


Fig. 3. (A) Plot of the difference of the phase velocity and $u_{\alpha 0}$ of the soliton against the ratio $U_{\alpha 0}/C$. Fixed $Q=6$, variable $n_{\beta 0}/n_{\alpha 0}$. (B) Plot of the difference of the phase velocity and $U_{\alpha 0}$ of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$. Fixed $Q=6$, variable $U_{\alpha 0}/C$. (C) Plot of the difference of the phase velocity and $u_{\alpha 0}$ of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$, variable Q.

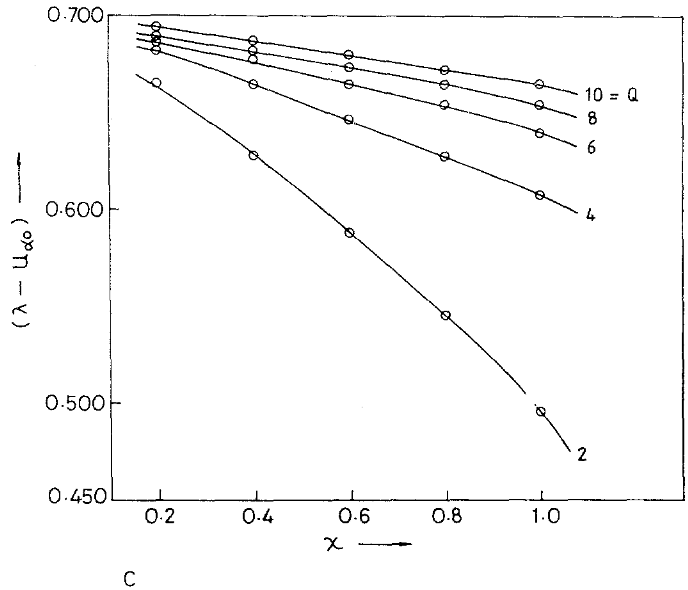


Fig. 3. Continued.

where

$$A_1 = \frac{15\omega}{8AK}$$

So the amplitude is proportional to A_1 and the width is given as $2(B/u)^{1/2}$. Figures 1A–1D depict the variation of the amplitude for various values of the streaming velocity and ratio of the ion densities. The width of the solitary wave is plotted in Figs. 2A–2C, for the same range of values of the parameters. Figures 3A–3C display the variation of the phase velocity. It is interesting to note that such a behavior of the solitary wave is very much possible in a nonisothermal plasma consisting of He^+ in the presence of negative ions such as H^- or O_2^-. A similar solution may occur in the case of a mixture of He^+ and N_2^-.

3. DISCUSSION

We have shown how a solitary wave may evolve in a nonisothermal plasma which is a mixture of positive and negative ions, with both ions having sufficient energy to be relativistic. Such situations already prevail in space plasmas and are of utmost importance for astrophysical considerations (Stenflo and Tsintsadze, 1979).

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